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Gamma-gamma angular correlations in Ca^{43}, Hg^{200} and Pb^{206}

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With 19 figures in the text

SUMMARY

An automatic apparatus for angular correlation measurements is described. The corrections for counter solid angle are experimentally determined. Gamma-gamma angular correlation measurements determine the spins of two excited levels in Ca^{43} and three excited levels in Hg^{200}. In Pb^{206} spins of five excited levels, given by Alburger and Pryce [21], are verified. Triple correlations are used in two cases.

Introduction

The measurement of the directional correlation (D.C.) between successive radiations is one of the methods of nuclear spectroscopy to determine spins and relative transition probabilities for excited states in a nucleus [1]. As soon as a nucleus decays through at least two successive radiations one can measure the D.C. between the two radiations, i.e. the angular distribution of the second radiation relative to the first radiation. The information which can be obtained from D.C. work depends, among other things, upon the type of radiation observed (α, β, γ, conversion electrons). The most important D.C. is the γ-γ correlation. Experimentally one measures the number of coincidences between the two successive radiations as a function of the angle θ subtended by the axes of the two counters (Fig. 1). Thus one obtains the correlation function $W(\theta)$, which can be formulated as

$$W(\theta)=\sum_{\nu} A_\nu P_\nu(\cos\theta) \quad (\nu=\text{even}),$$

where $P_\nu(\cos\theta)$ are the Legendre polynomials. The coefficients A_ν depend in the γ-γ-correlation on the spins of the three levels of the cascade and of the multipolarities of the radiations. If the cascade is denoted $j_1(L_1)j_2(L_2)j_3$ one has

$$\mathrm{A}_\nu=F_\nu(L_1L_1j_1j_2)\ F_\nu(L_2L_2j_2j_3).$$

The F_ν coefficients are tabulated, e.g. in ref. [1].

If one of the radiations consists of a mixture of different multipolarities, the γ-γ correlation determines very accurately the mixing ratio δ (δ^2 is the intensity ratio of the two multipolarities). The A_ν coefficients then take the form

$$A_\nu=\frac{1}{1+\delta^2}[F_\nu(L_1L_1j_1j_2)+2\delta\cdot F_\nu(L_1L_1'j_1j_2)+\delta^2\cdot F_\nu(L_1'L_1'j_1j_2)]\cdot F_\nu(L_2L_2j_2j_3),$$

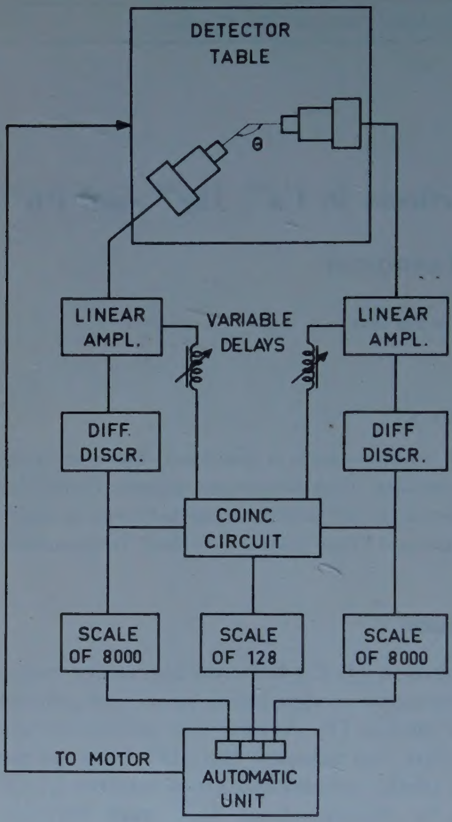

Fig. 1. Block diagram of angular correlatio apparatus.

where L_1 and L_1' are the multipolarities of the mixed transition. Very often tw different values of δ give the same correlation. A combination with conversion data however, excludes one of the two values.

The parities of the levels have no influence on the γ-γ D.C. In order to obtai information on parities other correlation experiments, as polarization or conversio electron correlation experiments, have to be carried out. It is also possible to combin conversion coefficient data with γ-γ-correlation data to determine parities.

So far it is assumed that the nucleus in the intermediate level is not disturbed b an extranuclear magnetic or electric field. When these interaction effects are present the correlation function is attenuated, in some cases completely wiped out to giv an isotropic distribution. If, however, the lifetime of the intermediate level is shorte than 10^{-10} s. or if the source is in a liquid form the interaction is too small to give an observable attenuation.

Apparatus

The apparatus used consisted of a fast-slow coincidence spectrometer (Fig. 1) Two detectors, one fixed and one movable, were mounted on a table (Fig. 2). Th movable detector position was changed with the help of a motor. NaI(Tl) crystals $1\frac{1}{2}$ in. diameter by $1\frac{1}{2}$ in. height, from Harshaw Chemical Corporation were couple

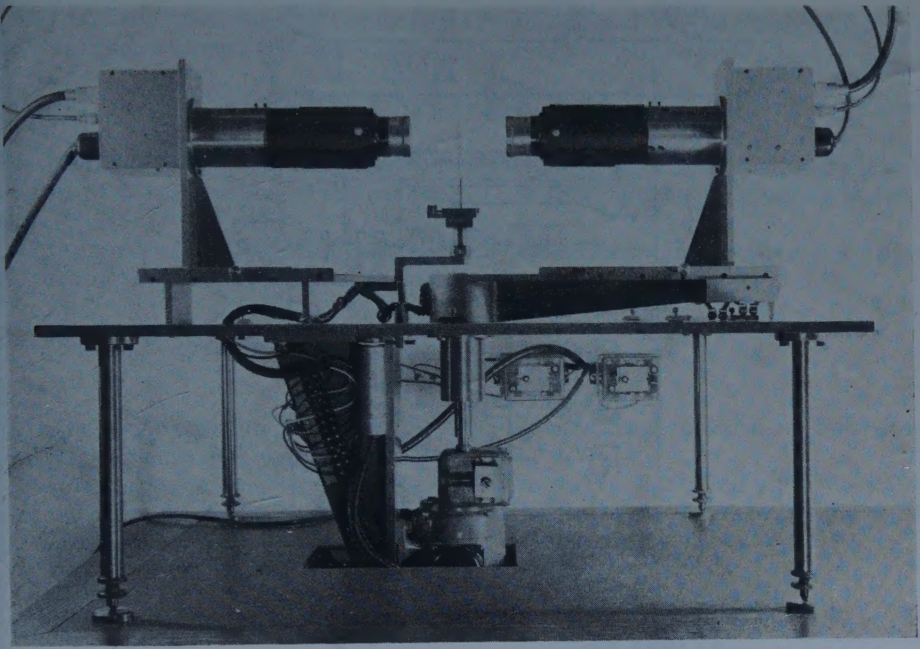

Fig. 2. Detector table. Movable detector to the right.

to RCA 6342 photomultipliers. The phototubes were covered by μ-metal in order to eliminate the influence of the earth magnetic field and of surrounding magnetic material. The photomultipliers were followed by linear amplifiers. From the linear amplifiers the pulses were fed via variable delays into a fast coincidence unit and were also fed parallel via differential discriminators into a slow coincidence unit. The fast-slow coincidence circuit had an effective resolving time of 1×10^{-7} s. The coincidence counting rate and the two single counting rates were recorded via scales onto mechanical registers. The apparatus was automatically run. The automatic unit[1] (Fig. 3) consisted of a synchronous motor that after a selected recording time shut off the scales for 10 seconds. During this pause the movable detector was shifted to a new position, the mechanical registers were photographed by a Robot camera and then reset to zero. The duration and position in time of the various operation pulses are shown in the diagram of Fig. 4.

The source, usually a small cylinder of 1 mm inner diameter and of 7 mm height, was mounted on a cross table (Fig. 2) in order to adjust the source in the centre within 1%, i.e. the counting rate of the movable detector should be constant within 1% for all positions.

Method of measurements

The relation between the number of coincidences $C(\theta)$ for a certain angle θ and the correlation function $W(\theta)$ is

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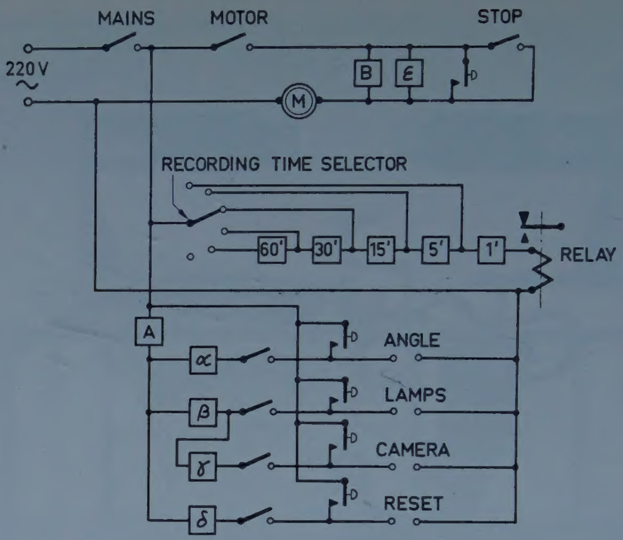
[1] The author is indebted to Dr. Ernst Heer, Zürich, for constructing the automatic unit.



A = CONTACT DURING PAUSE AND SHUT OFF FOR REGISTERS. (TWO CONNECTIONS ON THE RELAY)
B = CONTACT DURING RUN.
α = ANGLE SHIFT.
β = LAMPS FOR REGISTERS.
γ = CAMERA EXPOSURE
δ = RESET OF REGISTERS.
ε = STOP CONTACT.

Fig. 3. Circuit diagram of the automatic unit (constructed by Heer, Zürich).

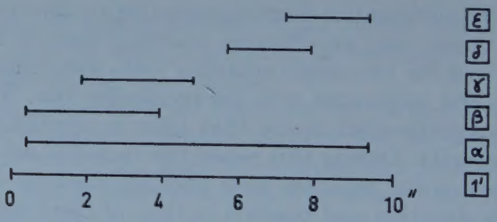


Fig. 4. Time diagram of the operation pulses in automatic unit.

$$C(\theta) = N_0 \varepsilon_1 \omega_1 \varepsilon_2 \omega_2 \varepsilon_c \cdot W(\theta) + C_a. \tag{1}$$

N_0 = disintegrations per second
$\varepsilon_{1,2}$ = efficiencies of resp. detectors
ε_c = efficiency of coincidence circuit (adjusted to 100%)
$\omega_{1,2}$ = solid angles of resp. detectors
C_a = accidental coincidences.

(A single cascade of unconverted gamma rays is assumed.)

The number of accidental coincidences is determined by the resolving time τ of the coincidence circuit and the number of counts in each channel.

$$C_a = 2\tau N_1 N_2 = 2\tau N_0 \varepsilon_1 \omega_1 N_0 \varepsilon_2 \omega_2. \tag{2}$$

The resolving time τ is measured once or twice a day during a run by inserting a delay of 0.5 μs. in one of the channels. C_a is then calculated according to the formula (2).

The quantity $C(\theta) - C_a$ is proportional to $W(\theta)$ and depending on the source strength N_0. One therefore forms the coincidence rate $CR(\theta)$.

$$CR(\theta) = \frac{C(\theta) - C_a}{N_2} = \varepsilon_1 \omega_1 \varepsilon_c \cdot W(\theta) = \text{const. } W(\theta). \tag{3}$$

N_2 is the single counting rate of the movable detector. Thus the error originating from a slightly decentred source is eliminated.

The numbers of coincidences $C(\theta)$ were recorded at 225°, 180°, 135° and 90°, as well as the single counting rates at corresponding positions. After correcting the data for accidental coincidences two quantities were formed [2], namely

$$A = \frac{CR(180) - CR(90)}{CR(90)}; \qquad U = \frac{CR(135) - CR(90)}{CR(90)}. \tag{4}$$

The coefficients can then be written as

$$A_2' = \frac{10(5A + 4U)}{7A + 56U + 105}; \qquad A_4' = \frac{48(A - 2U)}{7A + 56U + 105}. \tag{5}$$

Thus obtained coefficients A_2' and A_4' must be corrected for various effects in order to get the theoretical coefficients A_2 and A_4 in the correlation function

$$W(\theta) = 1 + A_2 P_2 (\cos\theta) + A_4 P_4 (\cos\theta).$$

These corrections will be treated below.

The statistical errors of the coefficients can be found by a straight forward method.

Corrections

1. *Solid angle*

The coefficients A_ν' have to be corrected for the finite solid angle of the counters. To determine the correction factors various methods are possible. The method used in this work was first given by Lawson and Frauenfelder [3]. Gamma rays were collimated by a 1 mm diameter hole through a lead cylinder, 10 cm diameter and 10 cm length (Fig. 5). The angular resolution curve $\varepsilon(\theta)$ for various energies was measured. The integrals

$$J_\nu = \int_0^{\theta_{\max}} P_\nu (\cos\theta)\, \varepsilon(\theta) \sin\theta \, d\theta \tag{6}$$

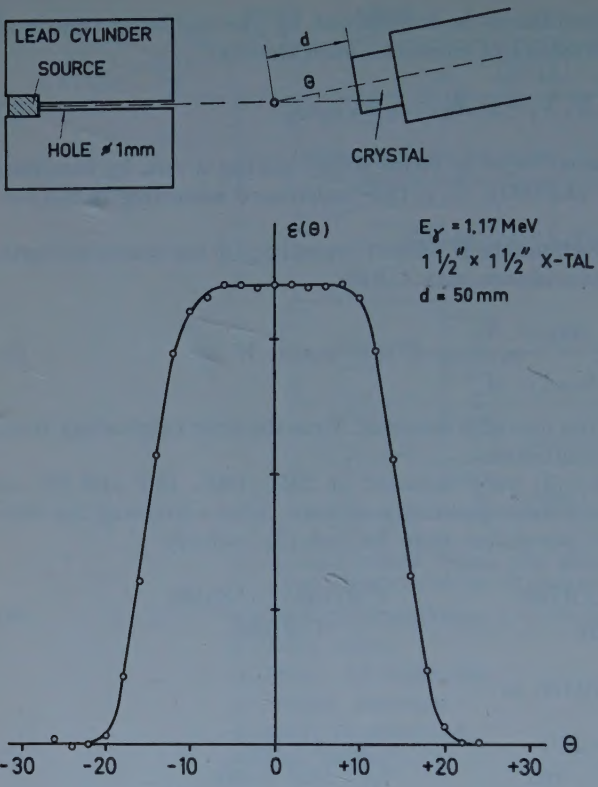


Fig. 5. Angular resolution curve and experimental set up.

were evaluated by numerical integration for $\nu = 0$, 2 and 4, using the observed angular resolution curves $\varepsilon(\theta)$ (Fig. 5). The correction factor s_ν for each detector is then

$$s_\nu = J_\nu / J_0 \,. \tag{7}$$

The energies investigated were 511 keV, 661 keV, 1.17 MeV, 1.33 MeV. s_ν as a function of energy is shown in Fig. 6, where also the theoretical value for the gamma energy $E = 0$ is shown. $s_{\nu,\,E=0}$ is easily calculated, because $\varepsilon(\theta)$ is then a constant. Both detectors gave identical results. The correction factors for the coefficients A_ν' are then the product of the factors for each detector

$$A_{\nu,\,\text{corr.}} = \frac{A_\nu'}{s_{\nu,\,E_1}\, s_{\nu,\,E_2}} \,. \tag{8}$$

2. *Source size*

In most correlation experiments the source can be so chosen that no correction for source size is necessary. The only source size correction which has been treated [4] is that for an axial source, i.e. a line source located in the centre and perpendicular to the plane of the two counters. This correction, applied to a source of 7 mm height on a distance of 50 mm to the 1½ in. diameter crystal, is of the order of a few per cent and can in most cases be neglected.

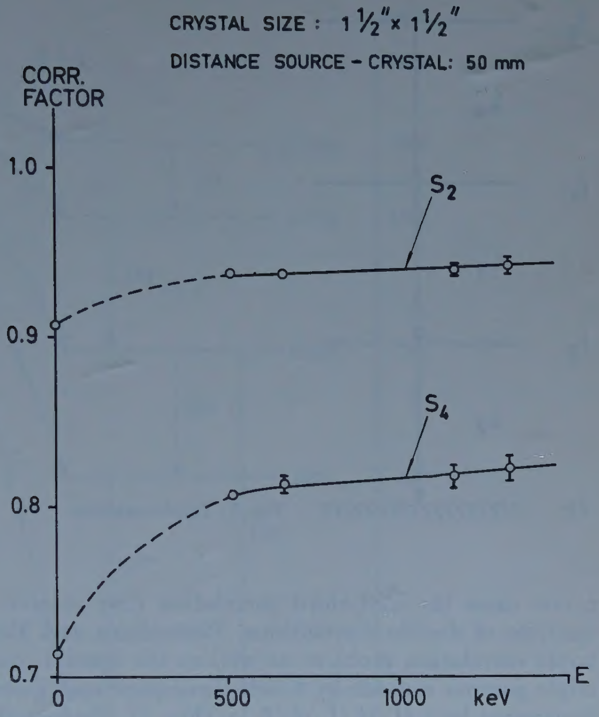


Fig. 6. Angular resolution corrections as a function of energy.

3. *Scattering*

The problem of correcting for scattering in the source is more complex and more difficult to solve than the others. However, using thin sources surrounded by material with low atomic number Z the scattering effect is very small. Further, if the γ-rays are of energies >200–300 keV, the scattering can be neglected.

Check on the apparatus

The whole apparatus was checked for correct working by determining the coefficients A_ν for the well known cascade in Ni^{60}. The 1.17–1.33 MeV gamma cascade occurs between spins 4, 2 and 0. The theoretically calculated coefficients [1, 5] for the cascade $4(Q)\,2(Q)\,0$ is $A_2 = +0.1020$ and $A_4 = +0.0091$. The experimentally determined coefficients after correction for counter solid angle and source size were

$$A_2 = +0.101 \pm 0.006 \quad \text{and} \quad A_4 = +0.008 \pm 0.010.$$

Triple correlations

There are several cases reported in the literature [6], in which there are three γ-radiations observed in cascade. The correlation between the first and the third γ-radiation will often be of importance. This correlation, sometimes called first-third correlation, may present no greater difficulty than the usual double correlation. In

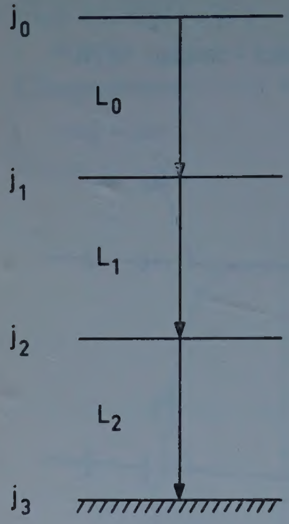


Fig. 7. Triple cascade.

a few cases the first-third correlation may resolve ambiguities resulting from the analysis of double correlations. Biedenharn and Rose [7] have treated the general triple correlation problem as well as the special case of first-third correlation for a triple gamma cascade in which three pure multipoles are emitted. If the cascade is designated by $j_0(L_0)j_1(L_1)j_2(L_2)j_3$ (Fig. 7), the first-third correlation function assumes the simple form

$$W(\theta)=N\sum_{\nu}F_{\nu}(L_0\,L_0\,j_0\,j_1)\,F_{\nu}(L_2\,L_2\,j_3\,j_2)\,W(j_1\,j_1\,j_2\,j_2;\,\nu\,L_1)\,P_{\nu}(\cos\theta), \tag{9}$$

where $$N=(-1)^{L_1-j_1-j_2}[(2j_1+1)(2j_2+1)]^{\frac{1}{2}}.$$

The F-coefficients are tabulated [5, 7] and $W(j_1 j_1 j_2 j_2;\,\nu L_1)$ are the Racah coefficients [8]. The extension to triple correlations with mixed multipoles is straight forward. Thus, if the first transition is mixed with multipolarities L_0 and $L_0'=L_0+1$, one obtains

$$W(\theta)=N\sum_{\nu}[F_{\nu}(L_0\,L_0\,j_0\,j_1)+2\delta\cdot F_{\nu}(L_0\,L_0'\,j_0\,j_1)+\delta^2\cdot F_{\nu}(L_0'\,L_0'\,j_0\,j_1)]\times$$
$$\times F_{\nu}(L_2\,L_2\,j_3\,j_2)\times W(j_1\,j_1\,j_2\,j_2;\,\nu\,L_1)\;P_{\nu}(\cos\theta), \tag{10}$$

where δ^2 is the intensity ratio of L_0' multipole to the L_0 multipole.

Finally, we consider that the second unobserved transition is mixed. We designate the correlation function for pure multipoles by $W(L_0)$, where L_0 is the multipolarity index of the unobserved radiation. For the case in which this is a mixture of multipoles L_0 and L_0' the correlation function is simply [9]

$$W=\frac{1}{1+\delta^2}[W(L_0)+\delta^2\cdot W(L_0')]. \tag{11}$$

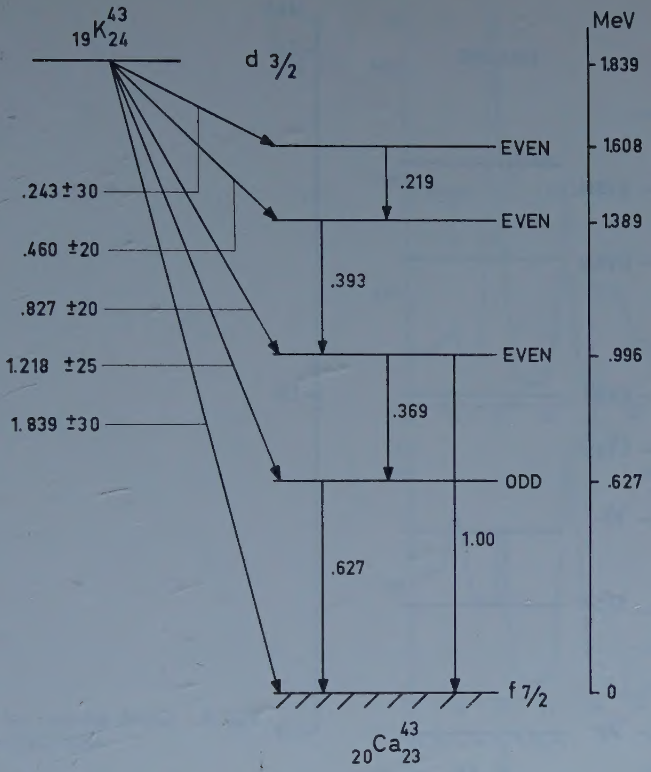


Fig. 8. Decay scheme of K^{43} (ref. [10]).

Levels in Ca^{43}

1. *Decay of K^{43}*

The decay of K^{43} ($T_{1/2} = 22.0$ h) to Ca^{43} has been investigated by Lindqvist and Mitchell [10], who reported five beta groups of the energies 1.839, 1.218, 0.827, 0.460 and 0.243 MeV and five gamma lines having energies 369, 627, 219, 393 keV and 1.00 MeV. The energy differences between the beta groups and concidences between 369 keV and 627 keV transitions made these authors propose the level scheme of Fig. 8. A following experiment on the decay of Sc^{43} to Ca^{43} indicated [11] that there also existed a level at 369 keV above the ground state. The levels in Ca^{43} thus found (Fig. 9) agreed within error limits with a (d, p) experiment on Ca^{42} by Braams [12]. However, refined experiments by Braams made it doubtful whether the gamma lines in the decay of K^{43} were put in the right order by Lindqvist and Mitchell. The level energies given by Braams [13] were 373, 593, 991, 1394 and 1678 keV (Fig. 9).

Recently Bäckström and Lindqvist [14] carried out an experiment involving redetermination of the gamma energies with a precision spectrometer and γ-γ-coincidence experiments. K^{43} was obtained as a spallation product from bombarding vanadium with 190 MeV protons [15] in the synchrocyclotron of the Werner Institute, Uppsala. A considerable amount of K^{42}($T_{1/2} = 12$ h) was simultaneously produced.

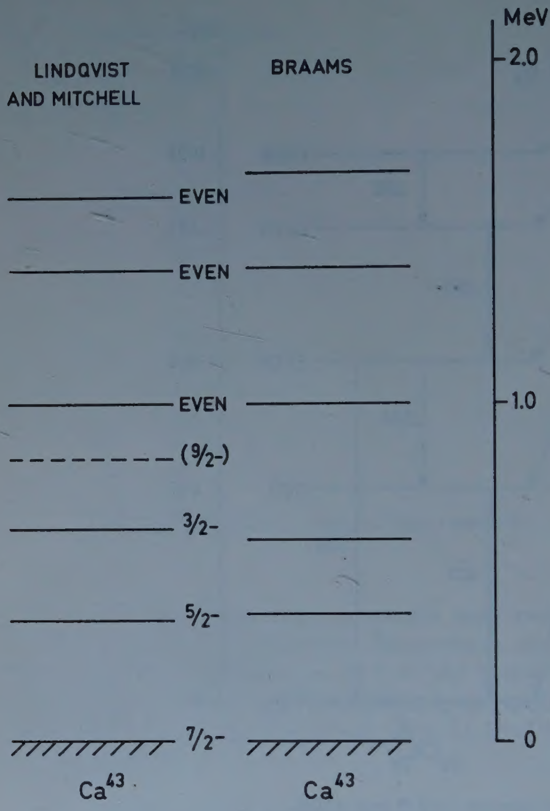


Fig. 9. Level scheme of Ca^{43} (ref. [11] and [12]).

The target was dissolved in nitric acid containing potassium carrier. Iron carrier was added, and the solution was made ammoniacal to precipitate the hydroxides. The vanadium was oxidized and precipitated with lead. The residue was strongly acidified with nitric acid and hydrochloric acid and then evaporated to dryness in a porcelain crucible. All ammonium salts were fumed off. The residue was dissolved in 1 ml of water. The perchlorate was precipitated with perchloric acid and alcohol. The potassium perchlorate was then washed well with absolute alcohol and dried. In this form it was used in the measurements of the gamma energies. For the angular correlation experiment an aqueous solution of potassium perchlorate was used.

The energy measurement of the two strongest gamma lines was carried out in a double focusing beta spectrometer ($r=50$ cm) by recording the photo lines from a 14 mg/cm^2 lead foil. By using the 560.5 keV line of As^{76} as a standard [16] the energies were determined as

$$\gamma_1=373.7\pm0.4 \text{ keV}, \qquad \gamma_2=618.9\pm0.6 \text{ keV}.$$

From the point of view of energies there was no doubt that, in the level scheme due to Braams, these transitions should be placed as a cascade leading to the ground state.

In a carefully calibrated scintillation spectrometer the high energy gamma line was determined to 1.02 ± 0.01 MeV. It can accordingly be concluded that this line is not

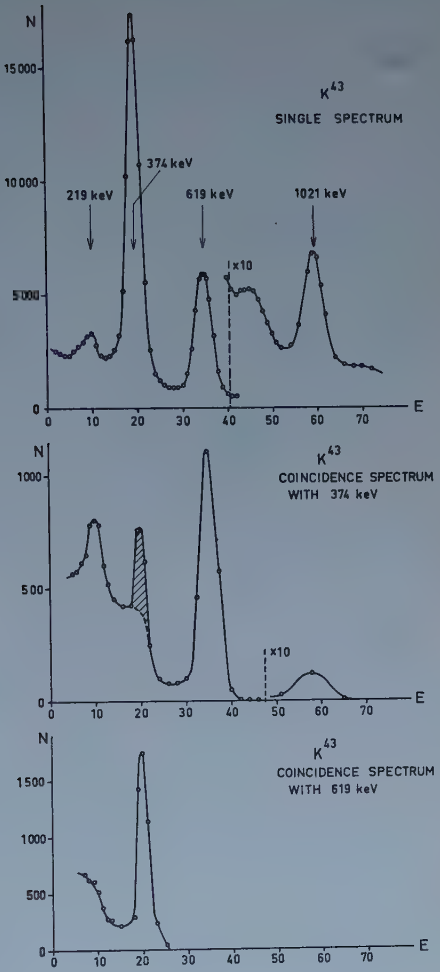


Fig. 10. Gamma spectrum of K^{43}. From top (a) single spectrum, (b) in coincidence with 374 keV line, and (c) in coincidence with 619 keV line.

a cross over to the 619 and 374 keV lines. The weak 393 keV line was not recorded in the beta-spectrometer, its intensity being too low and not in the scintillation spectrometer because of the energy which was too close to the strong 374 line. The weak 219 keV line was found in the coincidence experiment (see below) but the energy could not be accurately measured.

The coincidence experiments were carried out with a conventional coincidence scintillation spectrometer having an effective resolving time of 1×10^{-7} s. In Fig. 10 are shown the single gamma spectrum, the spectrum in coincidence with 374 keV and the spectrum in coincidence with 619 keV. The 1.02 MeV, 619 keV and 219 keV are in coincidence with the 374 keV line. The peak at 374 keV in the diagram *b* is the wellknown effect of the coincidence efficiency increasing a factor of two, when the scanning channel accepts the same energy as the fixed channel. The 619 keV line is not in coincidence with 219 keV or 1.02 MeV lines.

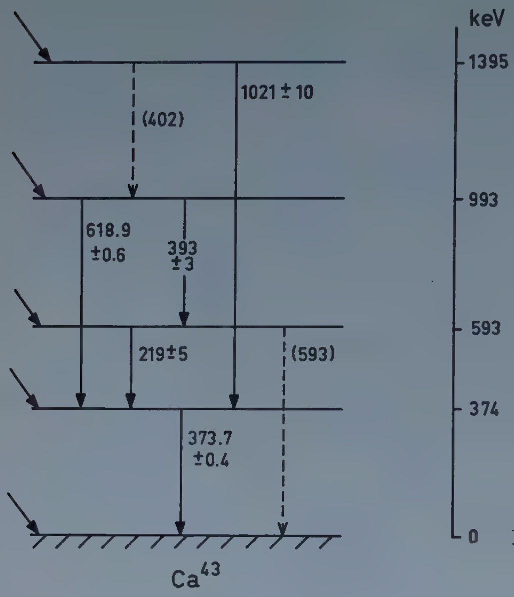


Fig. 11. Level scheme of Ca^{43} (ref. [14]).

Accordingly Bäckström and Lindqvist put the gamma lines into the level scheme of Ca^{43} as shown in Fig. 11, in good agreement with a recent experiment by Benczer-Koller *et al.* [17].

The spin of the ground state of Ca^{43} has been measured by Jeffries [18], who gives a spin of 7/2 and odd parity. Lindqvist and Mitchell have shown that the spin of the 374 keV level is $5/2^-$, and that of the 593 keV level is $3/2^-$. Ca^{43} has 3 neutrons outside the closed shell at 20 neutrons. It is reasonable to consider that the low-lying, odd levels arise from the $(f_{7/2})^3$ configuration. The excited states at 993 keV and 1395 keV were given even parity and could be explained by assuming a neutron from a lower shell being excited, thus giving a $(f_{7/2})^4$ + a neutron hole in the preceding shell. Therefore the even states can have the spins 1/2, 3/2, 5/2 according to the states in the previous shell, namely $s_{1/2}$, $d_{3/2}$ and $d_{5/2}$.

2. *Correlation experiments*

In order to check these spin data, correlation experiments were carried out. Two cascades in the level scheme of Ca^{43} were investigated, namely

$$W_{\mathrm{I}}(\theta) = 1021\text{–}374 \text{ keV},$$
$$W_{\mathrm{II}}(\theta) = 619\text{–}374 \text{ keV}.$$

The channels of the differential discriminators accepted respective photopeaks in each case. Contributions from other cascades could be neglected by reason of their low intensities. The results were after correction for counter solid angle:

$$W_{\mathrm{I}}(\theta) = 1 + (0.009 \pm 0.015)\, P_2(\cos\theta) + (0.034 \pm 0.038)\, P_4(\cos\theta),$$
$$W_{\mathrm{II}}(\theta) = 1 + (0.033 \pm 0.005)\, P_2(\cos\theta) + (0 \pm 0.009)\, P_4(\cos\theta).$$

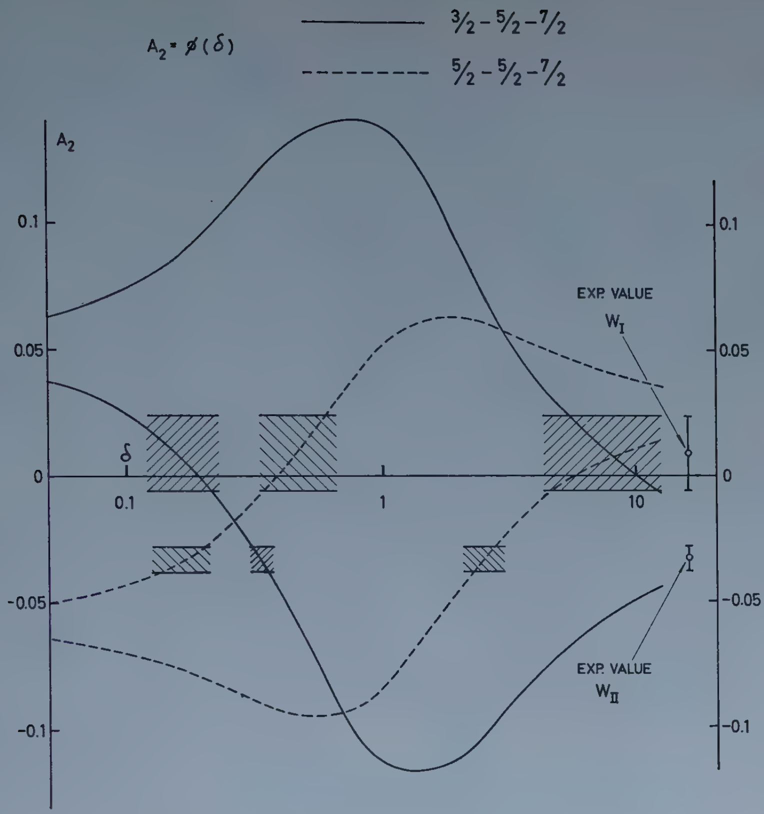


Fig. 12. A_2 as function of the mixing parameter δ for the spin sequences 3/2–5/2–7/2 and 5/2–5/2–7/2.

The two cascades have the 374 keV transition in common. The spin sequence for both cascades can accordingly be described as I–5/2–7/2. Within error limits both correlation functions have $A_4 = 0$. This indicates that the 374 keV transition between spins of 5/2 and 7/2 is a pure $M1$. The 1021 and 619 keV transitions can both be mixed by two different multipoles. Of the possible I-values $I = 1/2$, i.e. the spin sequence 1/2–5/2–7/2, can be excluded, because the measured correlations do not fit for any multipole admixture. In Fig. 12 are drawn the A_2-coefficients as a function of the mixing ratio δ for the spin sequences 3/2–5/2–7/2 (full line) and 5/2–5/2–7/2 (dashed line) as well as the experimental results. The measured correlations do fit both spin sequences. Therefore one cannot determine from correlation experiments which one of the spins 3/2 and 5/2 that belongs to the 993 keV level and the 1395 keV level, respectively. If the assumption about the neutron excitation from lower shell is correct the spin of 3/2 should belong to the lower energy because of the observed order of filling in the previous shell ($d_{5/2}$, $s_{1/2}$, $d_{3/2}$). Thus the spin of the 993 keV level is 3/2+ and that of the 1395 keV level is 5/2+.

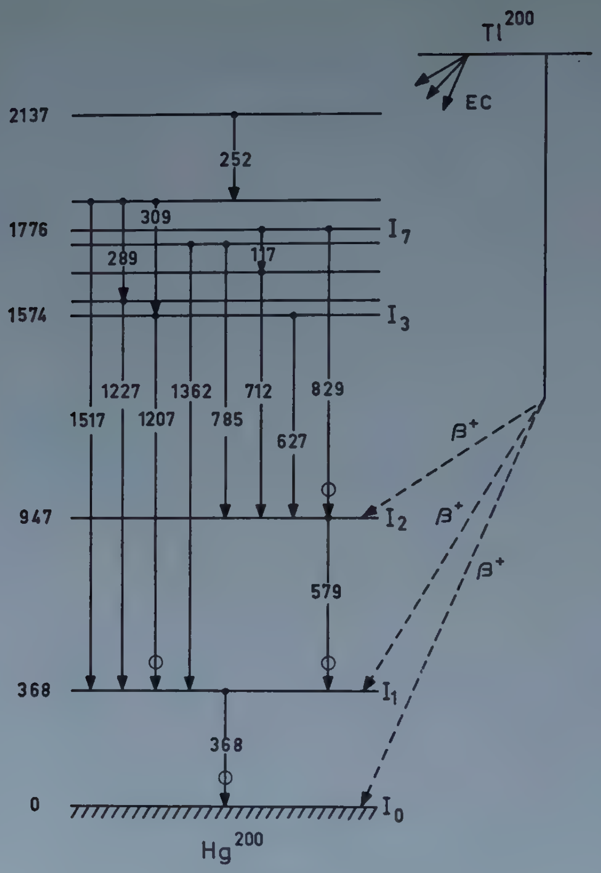


Fig. 13. Decay scheme of Tl^{200} (ref. [20]).

Levels in Hg^{200}

1. *Production and separation of* Tl^{200}

The thallium activity was obtained by milking thallium from active lead, produced by bombarding thallium with 65 MeV protons in the synchrocyclotron of the Werner Institute, Uppsala. The technique of separating lead from irradiated thallium is described in detail elsewhere [19]. The grown in thallium activity was extracted from lead after 35 hours. A solution of $TlCl_3$ was used for the correlation experiment.

2. *Decay of* Tl^{200}

The decay of Tl^{200} (Fig. 13) has been worked out by Herrlander and Gerholm [20]. Tl^{200} decays by electron capture to excited levels in Hg^{200}. The half-life of Tl^{200} is 27 h. The ground state of Hg^{200} has spin $I_0=0^+$ (even-even nucleus). From conversion data the first excited level was given a spin of 2^+ and the second excited level a spin

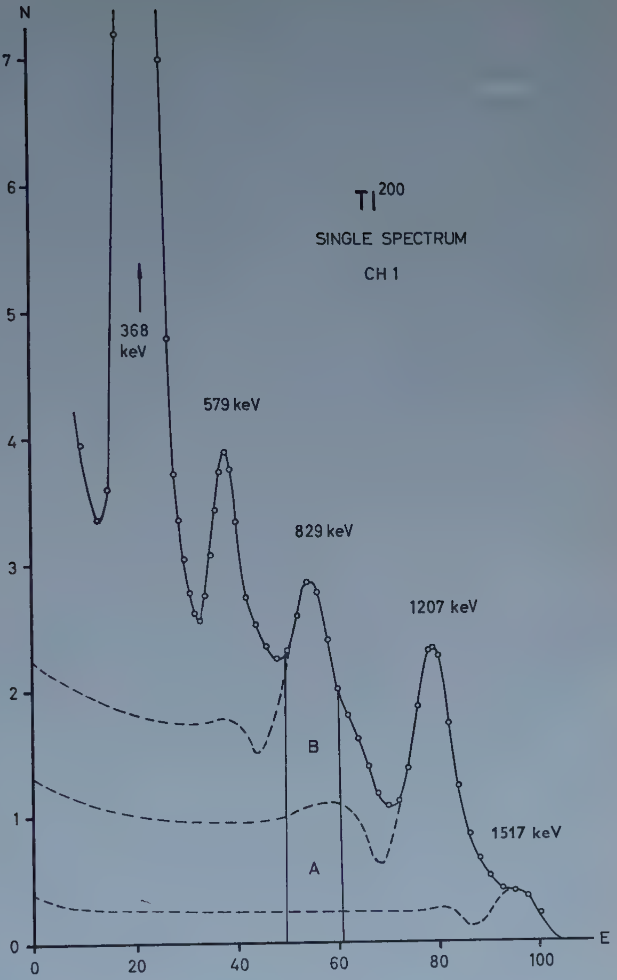


Fig. 14. Single gamma spectrum of Tl^{200}.

of 4^+ or 2^+. The absence of a crossover from the 947 keV level to the ground state gives preference to $I_2 = 4^+$. For other levels no definite spin assignments could be done.

3. *Correlation experiments*

Three gamma cascades in the level scheme of Hg^{200} were investigated by directional correlation measurements, namely

$$\begin{aligned} W_1(\theta) &= 1207 \text{ keV} - 368 \text{ keV}, \\ W_2(\theta) &= 829 \text{ keV} - (579 \text{ keV}) - 368 \text{ keV}, \\ W_3(\theta) &= 829 \text{ keV} - 579 \text{ keV}. \end{aligned}$$

The gamma rays concerned are marked with circles in the decay scheme (Fig. 13).

Combining information from the decay scheme and the single gamma spectrum (Fig. 14) one finds that $W_1(\theta)$ can be investigated directly by setting one channel on the 1207 keV photopeak and the other channel on the 368 keV photopeak. The contribution from the 1517–368 keV cascade can be neglected by reason of low intensity. The contribution from the 1227–368 keV cascade is probably small. If, however, the contribution is comparable with the 1207–368 keV cascade, the correlation function $W_1(\theta)$ is a sum of two correlation functions. This sum will always be the same for all settings of the differential discriminators.

$W_2(\theta)$, which is an example of a first-third correlation [7], must be measured in combination with $W_1(\theta)$. The relative contributions of the two cascades can be found from the analysis of the distribution of the gamma rays (Fig. 14). This analysis was made possible after calibration of the crystal with single gamma rays at the proper distance between source and crystal. If the measured combination of $W_1(\theta)$ and $W_2(\theta)$ is called $W_{12}(\theta)$, the following relation holds:

$$W_{12}(\theta)=\frac{1}{A+B}[A\cdot W_1(\theta)+B\cdot W_2(\theta)] \tag{12}$$

or

$$W_2(\theta)=\frac{1}{B}[(A+B)\cdot W_{12}(\theta)-A\cdot W_1(\theta)], \tag{13}$$

where the areas A and B (Fig. 14) are proportional to the contributions from the gamma rays concerned. $W_{12}(\theta)$ was measured by setting the discriminator channels on the 368 keV and 829 keV photopeaks, respectively.

$W_3(\theta)$ can be measured without any other cascades contributing.

The 579–368 keV cascade cannot be measured without other disturbing cascades. Below the photopeak of the 579 keV line one has the Compton distributions from the 829 keV and 1207 keV lines, which both contribute to the coincidences. An analysis of the relative distribution in such a case is very inaccurate. Another drawback of this case is the fact that the 829–368 keV and the 579–368 keV cascades are not statistically independent, i.e. the final correlation is not a linear combination of the two correlations with the relative intensities as weighting factors. Instead, the final correlation is an example of a triple correlation with parallel radiation. According to Biedenharn and Rose [7] such a correlation is "clearly rather cumbersome and involves laborious calculation before numerical results can be obtained". It was decided not to treat the 579–368 keV cascade, because it gives no further information.

4. *Results*

The results are given in Table I, where W_1, W_2 and W_3 are corrected for counter solid angle.

Table I.

Corr.	$A_2 \pm$ stat. errors	$A_4 \pm$ stat. errors
$W_1(\theta)$	$+0.063 \pm 0.004$	$+0.029 \pm 0.008$
$W_{12}(\theta)$	-0.015 ± 0.010	$+0.029 \pm 0.015$
$W_2(\theta)$	-0.086 ± 0.030	$+0.062 \pm 0.040$
$W_3(\theta)$	-0.123 ± 0.007	$+0.070 \pm 0.015$

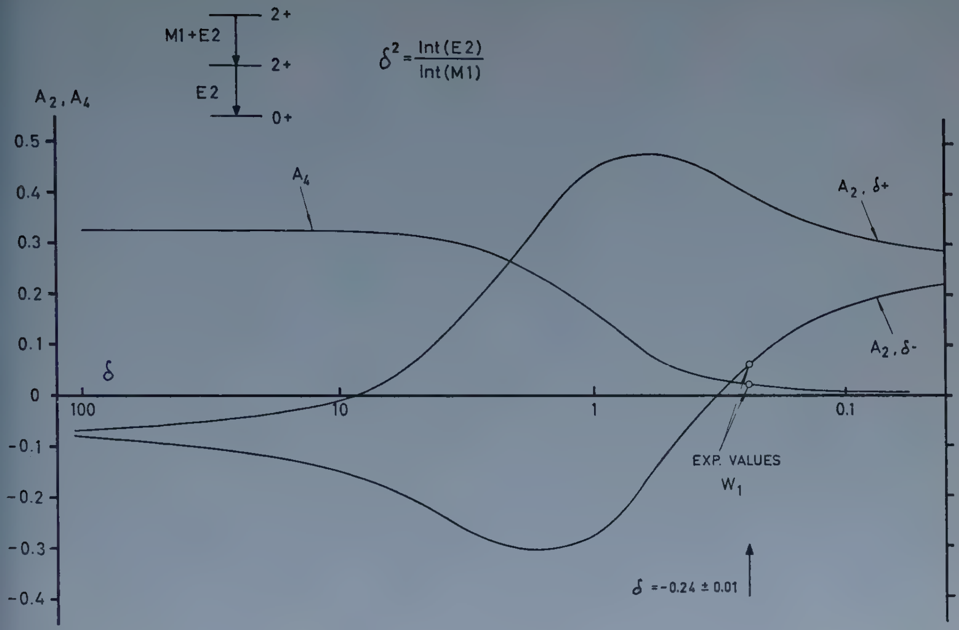


Fig. 15. A_2 and A_4 as functions of the mixing parameter δ for the spin sequence 2–2–0.

5. *The 1207–368 keV cascade*

The correlation function is

$$W_1(\theta) = 1 + (0.063 \pm 0.004)\, P_2(\cos\theta) + (0.029 \pm 0.008)\, P_4(\cos\theta).$$

The 368 keV transition is definitely $2^+(E2)\,0^+$. Assuming the 1227–368 keV cascade negligible, the only spin of the 1575 keV level that gives the measured correlation is $I_3 = 2$. In the spin sequence 2–2–0 the first transition is usually a dipole-quadrupole admixture. $A_\nu = \phi(\delta)$ for 2–2–0 (Fig. 15) indicates that $I_3 = 2$ requires $\delta = -0.24 \pm 0.01$, i.e. the 1207 keV transition should be 95% dipole +5% quadrupole. The parity cannot be determined. Comparing conversion data with the result $I_3 = 2$ one is apt to suggest even parity for the 1575 keV level.

6. *The 829–579–368 keV cascade*

This cascade was measured by two correlations. The first-third correlation function $W_2(\theta)$, 829–(579)–368 keV cascade, was

$$W_2(\theta) = 1 - (0.086 \pm 0.030)\, P_2(\cos\theta) + (0.062 \pm 0.040)\, P_4(\cos\theta).$$

The large errors are due to the estimated errors in the distribution analysis. The correlation function $W_3(\theta)$, 829–579 keV cascade, was

$$W_3(\theta) = 1 - (0.123 \pm 0.007)\, P_2(\cos\theta) + (0.070 \pm 0.015)\, P_4(\cos\theta).$$

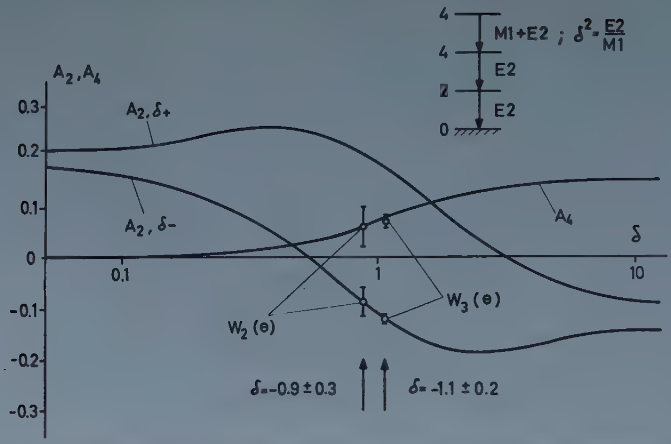


Fig. 16. A_2 and A_4 as functions of the mixing parameter δ for the spin sequence 4–4–2–0 and the spin sequence 4–4–2.

$W_2(\theta)$ and $W_3(\theta)$ are of course closely related, because they involve partly the same levels. The whole triple cascade can be described by the spin sequence

$$I_7(L_0L_0')\,I_2(L_1L_1')\,2\,(E2)\,0.$$

As mentioned above I_2 is either 4^+ or 2^+.

Let us first consider $I_2=2^+$. In this case the 579 keV transition is a dipole-quadrupole admixture. $W_2(\theta)$ is then described as (eq. 11)

$$W_2(\theta)=\frac{1}{1+\delta^2}[W(M1)+\delta^2\cdot W(E2)], \tag{14}$$

where δ is the 579 keV mixing ratio, and $W(M1)$ and $W(E2)$ are given by eq. (10). $W_3(\theta)$ is described as an ordinary double mixed correlation. After some elaborate calculations it turns out that $I_2=2^+$ does not give unique answers for the mixing ratio, whatever value I_7 is given.

If $I_2=4^+$, both the second and third radiations are pure $E2$. It turns out that $W_2(\theta)$ and $W_3(\theta)$ are described by the same correlation function. One has to try different values of I_7. The only one that gives unique answers for the mixing ratio is $I_7=4$. Even parity is most likely according to conversion data. The coefficients A_2 and A_4 as functions of the mixing ratio of the 829 keV transition is shown in Fig. 16. The experimental values of $W_2(\theta)$ and $W_3(\theta)$ are also given. The mixing ratios thus defined agree within the error limits. It is therefore concluded that $I_2=4^+$. The mean value of δ is 1.0 ± 0.1, i.e. the 829 keV transition is 50% $M1$ + 50% $E2$.

Levels in Pb^{206}

1. *Production and separation of Bi^{206}*

Bi^{206} was produced by bombarding lead with 16 MeV deutrons in the cyclotron at the Nobel Institute, Stockholm. The deutron energy was chosen to give no bismuth isotopes with $A<206$. The presence of Bi^{207} ($T_{1/2}=27$ y) was very small. Possible

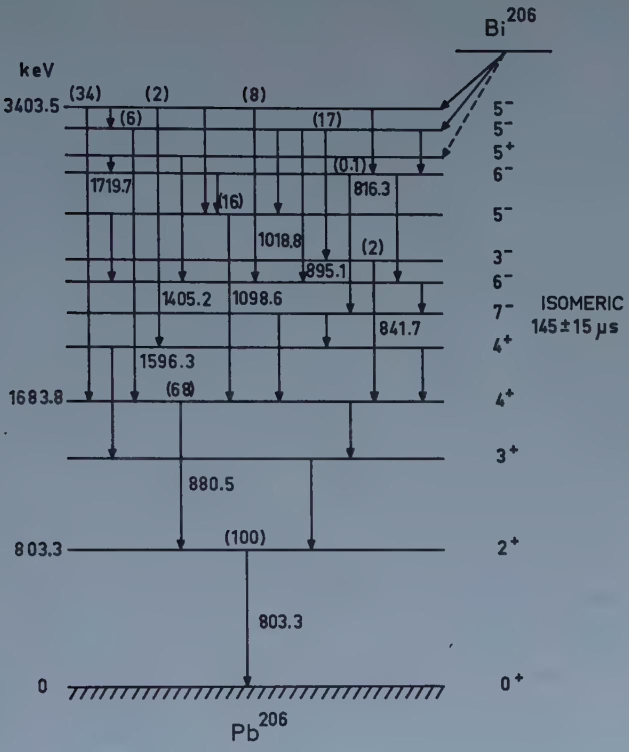


Fig. 17. Decay scheme of Bi^{206} (ref. [21]).

production of lead isotopes by (*d,pn*) reactions gave only short-lived isotopes. The bismuth activity was separated from most of the lead by precipitating lead chloride. A solution of the bismuth activity was put into a small cylinder and was used as source in the correlation experiment.

2. *Decay of Bi^{206}*

Alburger and Pryce [21], in a combined experimental and theoretical work, have very carefully determined the decay scheme of Bi^{206} (Fig. 17). Bi^{206} decays by electron capture to excited states in Pb^{206}. The half-life is 6.4 days. In Fig. 17 only gamma rays with energy >800 keV have been given energy and relative intensity. Spin assignments are given to all levels.

3. *Correlation experiments*

Because of the complexity of the gamma ray spectrum a clean correlation experiment is difficult to carry out. However, the 1720–880–803 keV cascade can be measured in the following way. One channel is set on the 1720 keV photopeak, well resolved from other lines (Fig. 18). The 880 keV and 803 keV lines are in coincidence with the 1720 keV line. A narrow window in the other channel is then set over various parts of the unresolved 880–803 keV lines (Fig. 19). For each setting the correlation

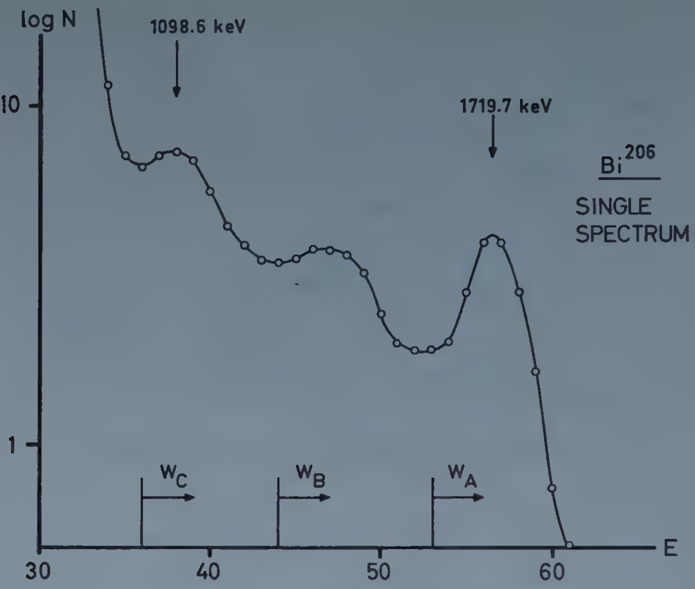


Fig. 18. Single gamma spectrum of Bi^{206} (high energy part).

is measured. The discriminator settings of the second channel are called I, II ... IV, according to Fig. 19, and the corresponding correlation functions are W_{I}, W_{II}, ... W_{IV}. In the case of W_{I} and W_{II} only the 1720–880 keV correlation is measured, in W_{IV} only the first-third 1720–803 keV correlation is measured, because the 880 keV line has its Compton "valley" in this position. W_{III} is a combination of W_1 (1720–880) and W_2 (1720–803).

4. *Results*

The results are given in Table II.

Table II.

Corr.	$A_2 \pm$ stat. errors	$A_4 \pm$ stat. errors
W_{I}	-0.052 ± 0.012	$+0.015 \pm 0.020$
W_{II}	-0.050 ± 0.010	$+0.008 \pm 0.015$
W_{III}	-0.063 ± 0.008	-0.004 ± 0.012
W_{IV}	-0.054 ± 0.008	$+0.002 \pm 0.012$

Within the error limits all measured correlations are identical. After correcting for counter solid angle the mean value of the measured coefficients are $A_2 = -0.067 \pm 0.008$ and $A_4 = +0.004 \pm 0.012$. This is in good agreement with the spin sequence given by Alburger and Pryce [21], namely 5^- ($E1$) 4^+ ($E2$) 2^+ ($E2$) 0^+. The two correlations W_1 (1720–880) and W_2 (1720–(880)–803) are identical for such a spin sequence. The theoretical values are $A_2 = -0.0715$ and $A_4 = 0$.

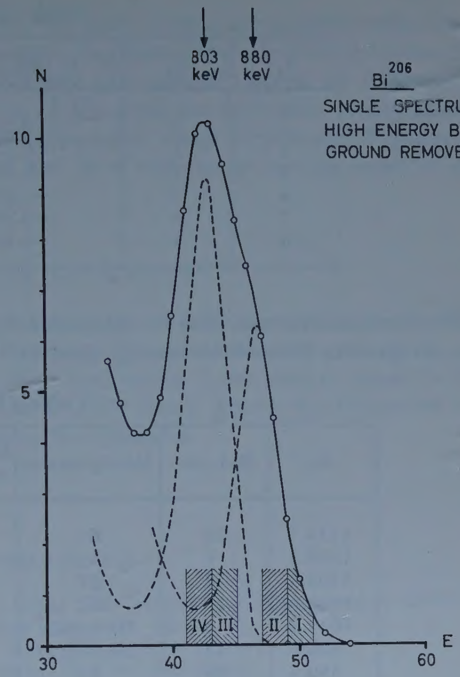


Fig. 19. Single gamma spectrum, 880–803 keV lines, Bi^{206}.

Under the assumption that the ground state is 0^+ and the first excited state is 2^+ one can show that the spin sequence $5^-(E1)\,4^+\,(E2)\,2^+\,(E2)\,0^+$ is the only one to give the measured results. The triple cascade 1720–880–803 keV is represented by the spins j_0, j_1, j_2, j_3 and the multipolarities L_0, L_1 and L_2 according to Fig. 7. In this case we have $j_3=0$, $j_2=2$ and $L_2=E2$. The first-third correlation 1720–803 keV is then (eq. 9)

$$W_2(\theta)=N\sum_{\nu}F_\nu(L_0L_0j_0j_1)\,F_\nu(E2,2,0)\times W(j_1j_1\,22;\ \nu L_1)\,P_\nu(\cos\theta) \tag{15}$$

and the double correlation 1720 − 880 keV

$$W_1(\theta)=\sum_{\nu}A_\nu P_\nu(\cos\theta)=\sum_{\nu}F_\nu(L_0L_0j_0j_1)\,F_\nu(L_1L_1j_1\,2)\,P_\nu(\cos\theta). \tag{16}$$

From the experiment it is concluded that $W_1(\theta)=W_2(\theta)$. Thus, for $\nu=2$, one obtains

$$N\cdot F_2(E2,2,0)\times W(j_1j_1\,22;\ 2L_1)=F_2(L_1L_1j_1\,2). \tag{17}$$

This relation is independent of j_0 and L_0. Only one parameter is disposable, namely j_1. L_1 is of course dependent on j_1 and $j_2=2$. In Table III is shown that eq. (17) is true only for $j_1=4$ and $L_1=2$.

Table III.

j_1	L_1	$N \cdot F_2(E2, 2, 0) \times W$	$F_2(L_1 L_1 j_1 2)$
1	1	-0.354	$+0.071$
2	1	-0.299	-0.418
3	1	-0.495	$+0.346$
4	2	-0.448	-0.448
5	3	-0.306	-0.736
6	4	-0.405	-0.872

Other correlations can also be measured by an indirect method. In Table IV are given all gamma lines with energy greater than 800 keV.

Table IV.

E_γ	Rel. int.	Multipolarity	In prompt coinc. with (880 + 803)	Origin level spin
1719.7	34	$E1$	yes	5^-
1596.3	6	$E1$	yes	5^-
1405.2	2	$E1$	(yes)[a]	5^-
1098.6	16	$E1$	yes	5^-
1018.8	8	$M1+(E2)$	no	5^-
895.1	17	$M1+(E2)$	no	5^-
880.5	68	$E2$	—	4^+
841.7	2	$E1$	yes	3^-
816.3	0.1	$M1+(E2)$	no	6^-
803.3	100	$E2$	—	2^+

[a] "(yes)" indicates that there is a transition link between the gamma line and the (880 + 803) lines.

From Table IV one can see that, if the spin assignments are correct, the correlation function should not change when the high energy channel went down to 1000 keV. All gamma lines in prompt coincidence with (880 + 803) lines originate from levels with spin 5^-. The 1405.2 keV line has a transition link to the (880 + 803) lines and gives another correlation. However, the relative intensity is too small to change the total correlation considerably. The 841.7 keV and 816.3 keV lines will not be detected with the discriminator set at 1000 keV.

Three different integral discriminator settings were used and the result is shown in Table V. The other channel accepted only the 880 keV photopeak.

Table V.

Corr.	Discr.	$A_2 \pm$ stat. errors	$A_4 \pm$ stat. errors
W_A	> 1600 keV	-0.051 ± 0.005	-0.002 ± 0.008
W_B	> 1200 keV	-0.051 ± 0.005	-0.002 ± 0.008
W_C	> 1000 keV	-0.058 ± 0.003	$+0.010 \pm 0.005$

A_ν not corrected for counter solid angle.

The three correlations, W_A, W_B and W_C, all agree within error limits with the earlier measured correlations, W_I–W_{IV}. One can therefore conclude that the spins of the levels 3403.5 keV, 3280.1 keV and 2782.9 keV are correctly given by Alburger and Pryce [21], namely 5^-. The A_4-coefficient of W_C does not include 0 within the error limits, indicating that the 1098.6 keV line has some small admixture of quadrupole radiation. Because of the complexity of the level scheme it has no sense to carry out a more quantitative analysis.

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